



Sovereign default and the decline in interest rates

Max Miller
HBS

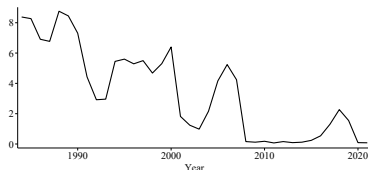
James D. Paron
Stanford

Jessica A. Wachter
Wharton

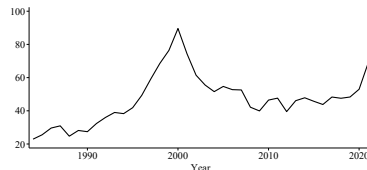


- Decline in bond yields 1980–2021, unmatched by rise in valuation ratios

Panel A: Federal funds rate



Panel B: Price-dividend ratio



- We examine these data through the lens of a standard asset pricing model.



- ▶ Aggregate endowment:

$$C_{t+1} = C_t e^{\mu} (1 - \chi_{t+1}),$$

where

$$\chi_{t+1} = \begin{cases} 0 & \text{with probability } 1 - p, \\ \eta & \text{with probability } p \end{cases}$$

- ▶ Representative investor, has constant relative risk aversion γ and elasticity of intertemporal substitution (EIS) ψ
- ▶ Discount factor $= \beta$.



- Price-dividend ratio:

$$\kappa = \frac{\beta e^{(1-\frac{1}{\psi})\mu} \left[1 + p((1-\eta)^{1-\gamma} - 1) \right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}}}{1 - \beta e^{(1-\frac{1}{\psi})\mu} \left[1 + p((1-\eta)^{1-\gamma} - 1) \right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}}}.$$

- Riskfree rate:

$$r_f \approx -\log \beta + \frac{1}{\psi} \mu - p((1-\eta)^{-\gamma} - 1) + \frac{\frac{1}{\psi} - \gamma}{1 - \gamma} p((1-\eta)^{1-\gamma} - 1).$$

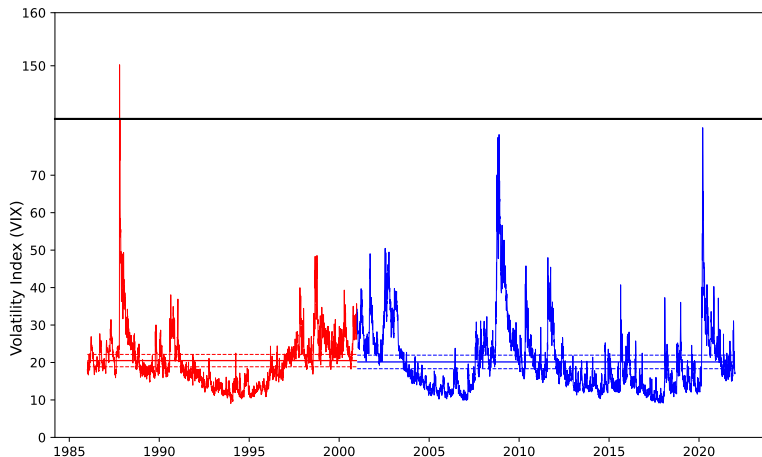
- Equity premium $\approx p\eta((1-\eta)^{-\gamma} - 1)$



	Values	
	1984–2000	2001–2016
Panel A: Data		
Price-dividend ratio	42.34	50.11
Inflation-adjusted Treasury yield	0.0279	-0.0035
Growth rate	0.0257	0.0157
Panel B: Model, $EIS = 2$		
Discount factor	0.969	0.982
Disaster probability	0.0225	0.0464
Panel C: Model, $EIS = 0.5$		
Discount factor	0.993	0.977
Disaster probability	0.0225	0.0464

► Risk aversion $\gamma = 12$, disaster size $\eta = 0.15$.

Did risk increase? Evidence from options





- ▶ Two versions:
 - ▶ Increase in patience (which should affect all assets)
 - ▶ A demand for assets that can also serve as currency
- ▶ Treasury securities, particularly short-term ones, often have a similar regulatory treatment as bank deposits.
 - ▶ Outside of their regulatory treatment, economic agents may have other reasons to treat them as cash.
- ▶ Thus they have a convenience yield:
- ▶ $y^b = R_f - \text{convenience}$.
- ▶ If the convenience term has increased, this would lead to a decrease in y^b , but R^f would stay the same.



- ▶ The convenience yield is hard/impossible to measure
 - ▶ Same factors that explain variation in convenience also may explain variations in risk.
 - ▶ For example, information asymmetry may be correlated with genuine risk.
- ▶ Nonetheless:

Convenience yield proxy =

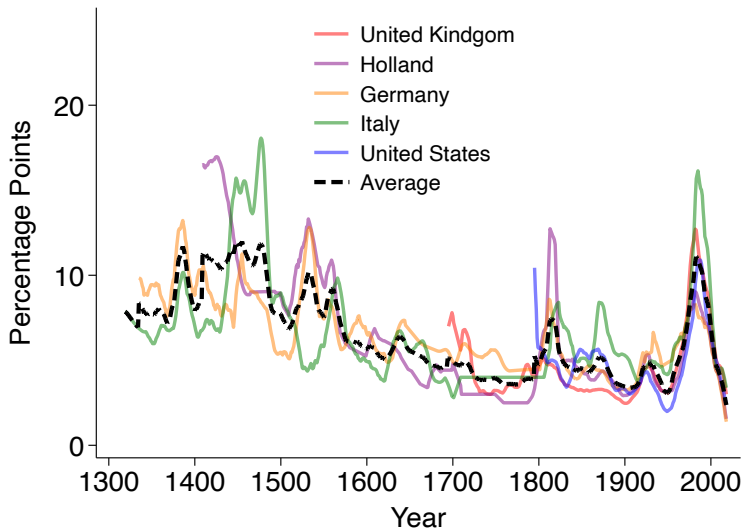
y^b – Yield on highest-rated corporate bonds

- ▶ Results: the convenience yield widened by 37 basis points between the two samples (\ll 300 basis point change in Treasury bonds).
- ▶ There may be some truth to the convenience yield mechanism, but it is not sufficient to explain the results.

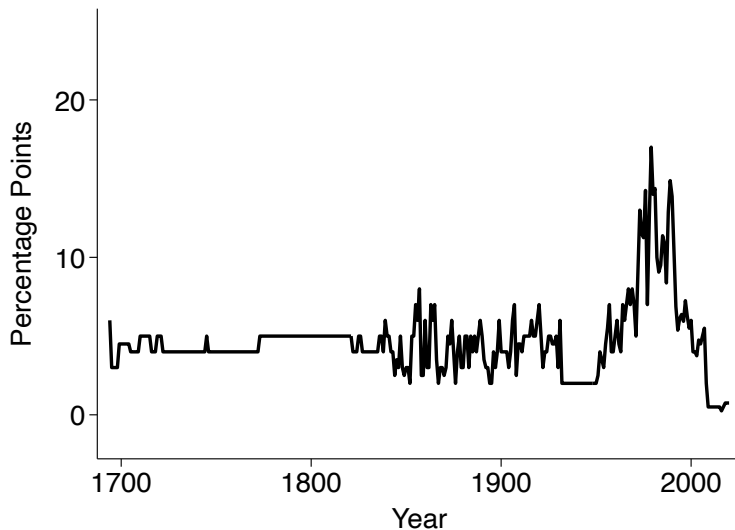


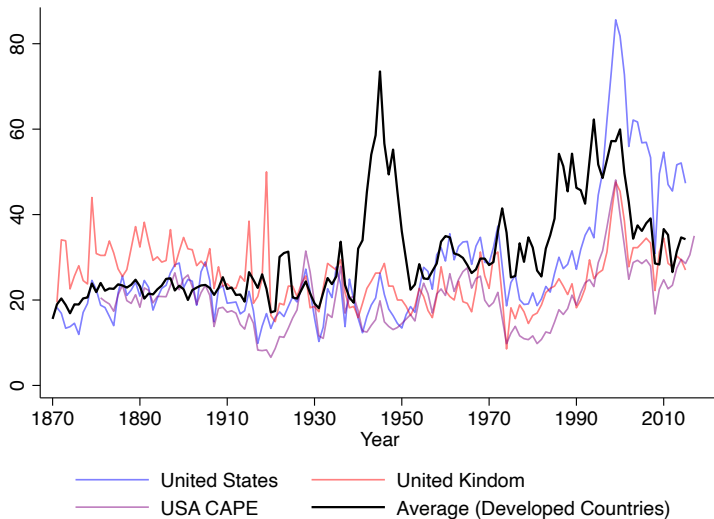
- ▶ Trust that governments will actually repay their debt has increased.
- ▶ Rather than the riskfree rate falling, Treasury bills (and other sovereign debt) have become more like the riskfree rate

Interest rate decline: a very-long term trend



Nominal Bank of England rate







- ▶ Bond payout: $L_t = \lambda \chi_{t+1}$

$$\chi_{t+1} = \begin{cases} 0 & \text{with probability } 1 - p, \\ \eta & \text{with probability } p \end{cases}$$

- ▶ Equilibrium condition:

$$\begin{aligned} y_b &\equiv -\log E_t [M_{t+1}(1 - L_{t+1})] \\ &\approx \log R_f + p\lambda\eta(1 - \eta)^{-\gamma} \\ &= \log R_f + p\lambda\eta + \underbrace{p\lambda\eta((1 - \eta)^{-\gamma} - 1)}_{\text{premium}} \end{aligned}$$

- ▶ The greater is the risk of default, the greater is y^b .
- ▶ Default can be outright or through inflation,



- ▶ Inflation varies during normal times, but spikes in times of disasters.
- ▶ These spikes (whether intentional or not) are priced and can be considered “default.”
- ▶ Assume all default takes place through inflation.



- Inflation

$$\Pi_{t+1} = \Pi_t e^{q_t + \sigma_\pi \epsilon_{t+1}} (1 - L_{t+1})^{-1},$$

- Nominal Treasury yield:

$$y_{b,t}^{\$} = -\log \mathbb{E}_t \left[M_{t+1} \frac{\Pi_t}{\Pi_{t+1}} \right].$$

Ex-post inflation adjusting gives y^b :

$$y^b \equiv \frac{1}{T - t_0} \sum_{t=t_0}^T y_{b,t}^{\$} - \Delta \pi_{t+1} = \frac{1}{T - t_0} \sum_{t=t_0}^T y_{b,t}^{\$} - (q_t + \sigma_\pi^\top \epsilon_{t+1})$$

- Nominal yield less ex post average inflation:

$$y_b = r_f + p\lambda\eta((1 - \eta)^{-\gamma} - 1) + p\lambda\eta$$

- A decline in λ captures a decline in inflation risk

		Values	
	Parameter	1984–2000	2001–2021
Panel A: Moments in the data			
Price-dividend ratio	κ	42.34	50.86
Inflation-adjusted Treasury yield	y_b	0.0279	-0.0069
Panel B: $\gamma = 5$, $EIS = 2$, $\eta = 0.3$			
Average consumption growth	μ	0.0257	0.0157
Discount factor	β	0.973	0.981
Fraction of bond value lost	$\lambda\eta$	0.129	-0.044
Model-implied riskfree rate	r_f	0.012	-0.002
Panel C: $\gamma = 5$, $EIS = 1$, $\eta = 0.3$			
Average consumption growth	μ	0.0257	0.0157
Discount factor	β	0.977	0.981
Fraction of bond value lost	$\lambda\eta$	0.129	-0.044
Model-implied riskfree rate	r_f	0.012	-0.002

► Risk aversion $\gamma = 5$, $EIS = 1$, disaster size $\eta = 0.30$ 

		Values	
	Parameter	1984–2000	2001–2021
Panel A: Moments in the data			
Price-dividend ratio	κ	42.34	50.86
Inflation-adjusted Treasury yield	y_b	0.0279	-0.0069
Panel B: $\gamma = 5$, $EIS = 2$, $\eta = 0.3$			
Average consumption growth	μ	0.0257	0.0157
Discount factor	β	0.973	0.981
Fraction of bond value lost	$\lambda\eta$	0.129	-0.044
Model-implied riskfree rate	r_f	0.012	-0.002
Panel C: $\gamma = 5$, $EIS = 1$, $\eta = 0.3$			
Average consumption growth	μ	0.0257	0.0157
Discount factor	β	0.977	0.981
Fraction of bond value lost	$\lambda\eta$	0.129	-0.044
Model-implied riskfree rate	r_f	0.012	-0.002

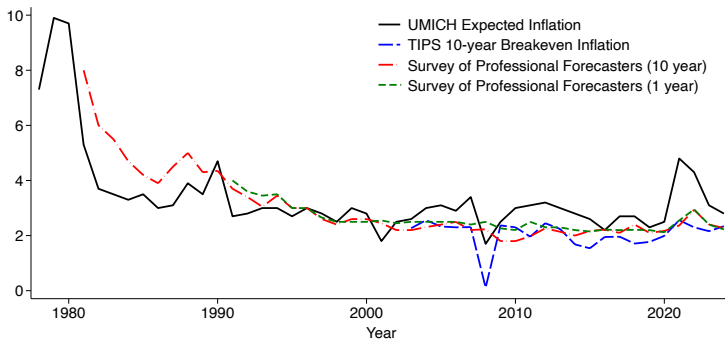
► Risk aversion $\gamma = 5$, $EIS = 1$, disaster size $\eta = 0.30$ 



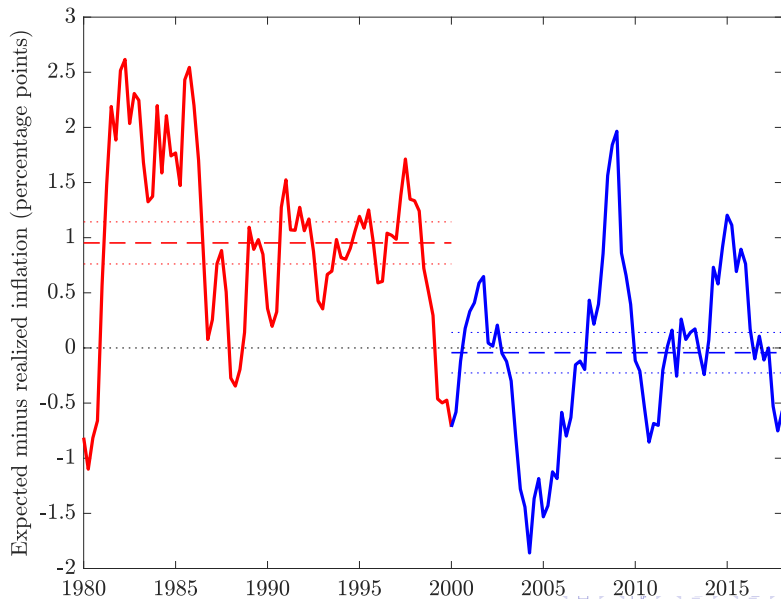
We look at

- ▶ Inflation expectations (informal)
- ▶ Inflation expectations versus realizations
- ▶ Inflation-consumption correlation
- ▶ Excess returns on nominal vs. inflation-linked Gilts

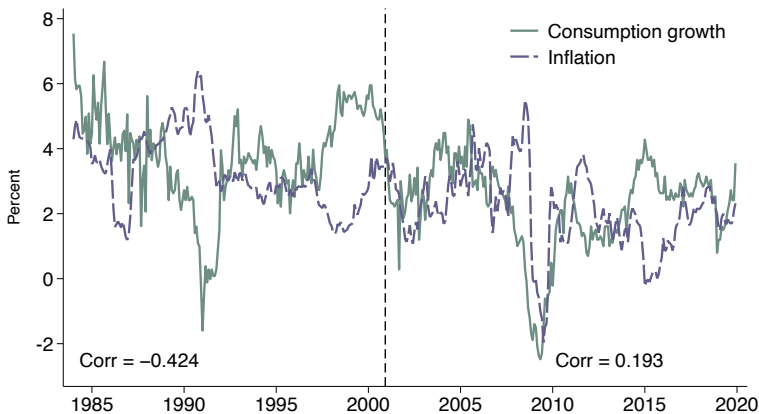
Inflation expectations



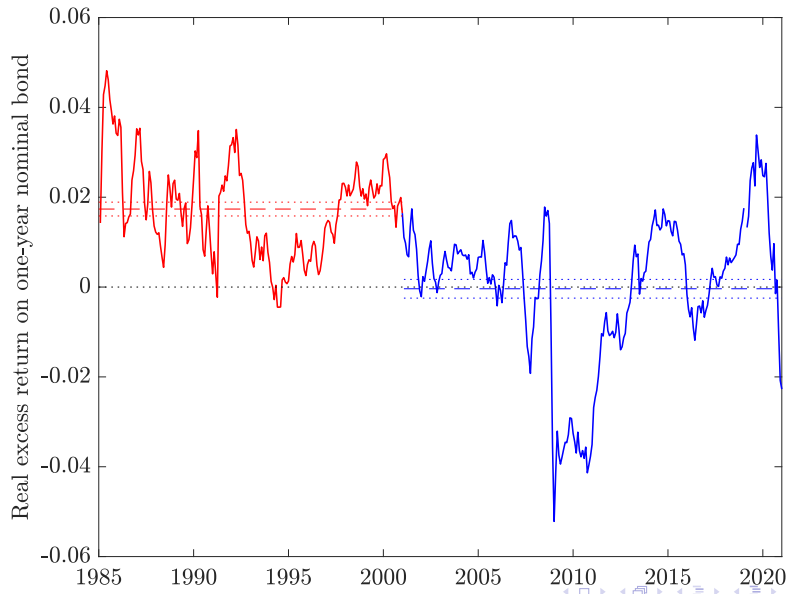
Inflation expectations versus realizations



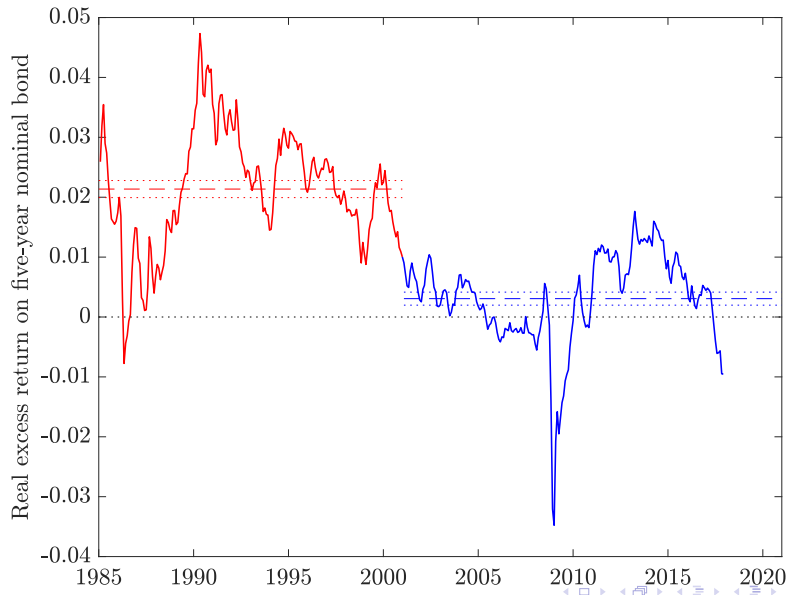
Inflation-consumption correlation



Excess returns on nominal vs. inflation-linked gilts



Excess returns on nominal vs. inflation-linked gilts





We can generalize the model to account for:

- ▶ Time-varying earnings expectations
- ▶ Time-varying inflation expectations that are not priced
- ▶ The change in correlation between inflation and consumption.
(consistent with though different from our mechanism).

This will allow us to isolate the effect of rare disasters (versus default expectations more broadly).

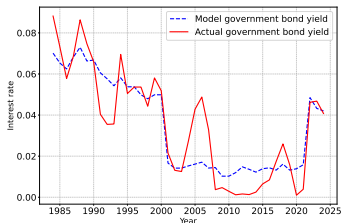


$z_t \equiv \log(C_t/D_t)$, the consumption-dividend ratio

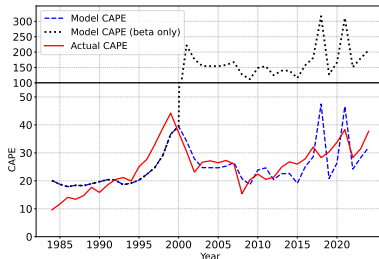
$$\begin{aligned}C_{t+1} &= C_t e^{\mu + \sigma_C^\top \varepsilon_{t+1}} (1 - \chi_{t+1}), \\z_{t+1} &= (1 - \rho_z) \bar{z} + \rho_z z_t + \sigma_z^\top \varepsilon_{t+1} - \xi \log(1 - \chi_{t+1}), \\\Pi_{t+1} &= \Pi_t e^{q_t + \sigma_\pi^\top \varepsilon_{t+1}} (1 - \lambda \chi_{t+1})^{-1}, \\q_{t+1} &= (1 - \rho_q) \bar{q} + \rho_q q_t + \sigma_q^\top \varepsilon_{t+1}.\end{aligned}$$

We proxy for q_t and z_t using survey data.

A. Interest rate



B. Valuation ratios



Note: dotted black line shows P/D were sovereign default to stay the same.

		Values		
	Parameter	1984–2000	2001–2021	2022–2024
Panel A: data				
Price-dividend ratio	κ	42.35	50.86	64.01
1-year Treasury yield	y_b	0.0593	0.0141	0.0446
Panel B: model				
Cons growth	μ	0.0257	0.0157	0.0157
Cons-inflation correlation	$\text{Corr}(C, \pi)$	-0.205	0.742	0.742
Discount factor	β	0.977	0.981	0.981
Fraction of bond value lost	$\lambda\eta$	0.106	-0.030	0.193

Notes: $\gamma = 5$, $\text{EIS} = 1$, $\eta = 0.3$



Interest rates increased dramatically in 2022. Why?

1. a reversal of the decline in β
2. an increase in expected growth μ
3. a decrease in the disaster probability p
4. an increase in expected inflation q_t
5. An increase in the risk of default during disasters λ_t .



1. A reversal of the decline in β would have caused a stock market crash, didn't happen
2. Little evidence for an increase in μ from survey data
3. No evidence for a decrease in p
4. Inflation expectations increased by a mere 1.5 percentage points
5. Only λ remains.



- ▶ Accounting for decline in interest rates and stability of valuation ratios requires an increase in macro risk for which there is no independent evidence.
- ▶ In contrast, a decline in sovereign default risk robustly explains the data and has independent support.
- ▶ An increase in sovereign default risk explains recent increases in rates